METHOD OF PROCESSING AND CORRELATION OF EXPERIMENTAL DATA FOR HEAT TRANSFER DURING CONDENSATION ON EXTERNAL SURFACES

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A general approach to the processing and correlation of experimental data, as exemplified by the condensation of a chemically reacting gas and an inert vapor–air mixture, is proposed.

The condensation of a chemically reacting gas, in which chemical reactions occur between the condensed and uncondensed components, is one of the most complex and, at the same time, most important practical thermophysical problems. A theoretical analysis of this problem entails the solution of an adjoint system of differential equations of heat and mass transfer in liquid and gas phases and equations of chemical kinetics. Such a solution for the chemically reacting gas system $N_2O_4 \rightleftharpoons 2NO_2 \rightleftharpoons 2NO + O_2$ was obtained in [1].

The results of calculations of the distributions of temperatures, velocities, concentrations, and other parameters during the condensation of a chemically reacting gas, given in [1], illustrate the mechanism of the process to some extent, but it is not at all obvious how these results can be used in practice. Firstly, the classical problem of condensation on a vertical surface was considered, whereas in real surface condensers condensation usually occurs on horizontal tubes. Secondly, in practical calculations of real condensers it is obviously essential to take into account the results of experimental investigations of the condensation of a chemically reacting gas in conditions which more or less simulate the process in real apparatuses.

These investigations were conducted in the Institute of High Temperatures, Academy of Sciences of the USSR [2], and the Institute of Nuclear Power, Academy of Sciences of the Belorussian SSR [3], but the only attempt to process these experimental data that has so far been published [2] is based on the use of the concept of the heat-transfer coefficient, which for the considered complex case of heat and mass transfer with physical and chemical changes, as was shown in [1], is a very arbitrary and simplified quantity.

Mikhalevich [4] developed a general approach to correlation of the results of numerical calculations of the condensation of pure vapor, vapor–gas mixtures, and a chemically reacting gas on a vertical surface, based on the method of "fractional" analysis of the differential equations and boundary conditions [5]. In the present work this method is used to process and correlate experimental data for the condensation of gas mixtures on horizontal tubes.

For the most general case of condensation of a chemically reacting gas the system of differential equations of heat and mass transfer in the adjacent gas boundary layer has the form

$$r\frac{\partial w_s}{\partial s} + \frac{\partial}{\partial r}(rw_r) = 0, \tag{1}$$

$$w_r\frac{\partial}{\partial r}(rw_r) + w_s r\frac{\partial w_s}{\partial s} = gr\left(\frac{\rho}{\rho_\infty} - 1\right) + \nu\left[\frac{\partial}{\partial r}\left(r\frac{\partial w_s}{\partial r}\right) - \frac{w_s}{r}\right], \tag{2}$$

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$$w_r \frac{\partial T}{\partial r} + w_s \frac{\partial T}{\partial s} = a \left[\frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) \right], \tag{3}$$

$$w_r \frac{\partial \rho_{k0}}{\partial r} + w_s \frac{\partial \rho_{k0}}{\partial s} = D_k \left[\frac{\partial}{\partial r} \left(r \frac{\partial \rho_{k0}}{\partial r} \right) \right] + \frac{I_k}{\rho}. \tag{4}$$

It is convenient to write Eq. (4) for the component O_2. In this case the expression for the mass source (sink) I_k has the form

$$I_k = -\frac{4K_{\text{rec}}}{M_4^2} \rho^3 \rho_{k0}^3. \tag{5}$$

For the indicated system the liquid phase can be regarded as a single substance and, thus, the diffusion equation can be excluded from the analysis. The boundary conditions for the two adjoint systems of differential equations in the gas and liquid phases of type (1)-(4) for the case of condensation on a horizontal tube have the following form:

on the tube surface ($r = \delta'$),

$$w'_s = w'_r = 0, \tag{6}$$

$$T' = T_0; \tag{7}$$

on the phase interface ($r = 0$),

$$w_s = w'_s, \tag{8}$$

$$\mu' \frac{\partial w'_s}{\partial r} = \mu \frac{\partial w_s}{\partial r}, \tag{9}$$

$$\rho' \left(w'_s \frac{d\delta'}{ds} - w'_r \right) = \rho \left(w_s \frac{d\delta}{ds} - w_r \right), \tag{10}$$

$$-\lambda' \frac{\partial T}{\partial r} = -\lambda \frac{\partial T}{\partial r} + \Delta h \rho \left(w_s \frac{d\delta'}{ds} - w_r \right), \tag{11}$$

$$\rho_{k0} \rho \left(w_s \frac{d\delta'}{ds} - w_r \right) = \rho D_k \frac{\partial \rho_{k0}}{\partial r}; \tag{12}$$

beyond the limits of the gas boundary layer (at infinity: $r \to \infty$),

$$w_s = 0, \tag{13}$$

$$T = T_\infty, \tag{14}$$

$$\rho_{k0} = \rho_{k0\infty}. \tag{15}$$

To problem (1)-(15) we apply the method of fractional analysis, for which the following two main requirements must be satisfied: All the equations and boundary conditions must be homogeneous as regards units of measurement; all the dimensionless variables which may be introduced in problem (1)-(15) must lie in the interval [0, 1] in the considered region.

It is easy to establish that the first of these requirements is automatically satisfied in the selected system of coordinates associated with the surface of the liquid film. To fulfill the second requirement we introduce new dimensionless variables:

$$\begin{gathered} \Phi = \frac{s}{\pi d_{\text{b}}}; \ \tilde{W}'_s = \frac{w'_s}{W'_s}; \ \tilde{W}'_r = \frac{w'_r}{W'_r}; \\ \tilde{T}' = \frac{T' - T_0}{T_s - T_0}; \ R' = \frac{r}{\delta'}; \ \tilde{W}_s = \frac{w_s}{W_s}; \ \tilde{W}_r = \frac{w_r}{W_r}; \\ \tilde{\rho}_{k0} = \frac{\rho_{k0} - \rho_{k0\infty}}{\rho_{k0\text{max}} - \rho_{k0\infty}}; \ \tilde{T} = \frac{T - T_\infty}{T_s - T_\infty}; \ R_{\text{т}} = \frac{r}{\delta_{\text{т}}}; \\ R_g = \frac{r}{\delta_g}; \ R_r = \frac{r}{\delta_{\text{h}}}. \end{gathered} \tag{16}$$

Substituting the new variables (16) in Eqs. (1)-(15), we obtain as coefficients dimensionless complexes which have the following form:

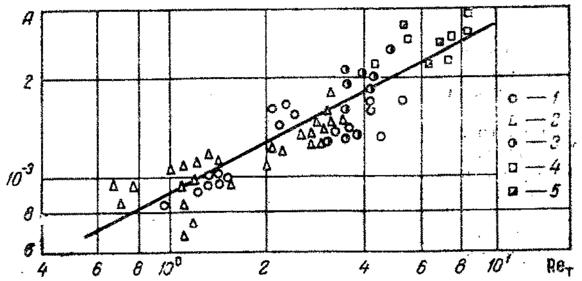

Fig. 1. Correlation of experimental data of [2] for condensation of a chemically reacting gas on a horizontal tube: 1) P = 2-3; 2) P < 2; 3) P = 3-4; 4) P = 4-5; 5) P = 6-8 atm.

in equations for the liquid phase of type (1)-(3),

$$\frac{W_r'\pi d_\mathrm{b}}{W_s'\delta'};\quad \frac{g\delta'\pi d_\mathrm{b}(\rho'-\rho)}{(W_s')^2\delta'\rho'};\quad \frac{\nu'\pi d_\mathrm{b}}{W_s'(\delta')^2};\quad \frac{a\pi d_\mathrm{b}}{W_s'(\delta')^2}; \tag{17}$$

in equations for the gas phase (1)-(4),

$$\frac{W_r\pi d_\mathrm{b}}{W_s\delta};\quad \mathrm{Sc}\,\frac{\delta_g}{\delta_\mathrm{h}};\quad \frac{g\delta_\mathrm{b}(\rho-\rho_\infty)}{\rho_\infty W_s W_r};\quad \frac{a\pi d_\mathrm{b}}{W_s\delta_\mathrm{T}^2};$$

$$\frac{D_k}{\delta_g W_r};\quad \frac{I_k\delta_g}{\rho\rho_{k0\infty}W_r}; \tag{18}$$

in boundary conditions (6)-(15),

$$\frac{\rho'}{\rho};\quad \frac{\mu'}{\mu};\quad \frac{\lambda'(T_s-T_0)}{\delta' w_r'\rho'\Delta h};\quad \frac{D_k}{\delta_g W_r}. \tag{19}$$

Using the estimation procedure devised in [4], we can use the above complexes to construct a dimensionless equation representing the experimental mass-transfer data for condensation of complex vapor–gas mixtures:

$$\mathrm{Nu}_D = \mathrm{const}\left(\frac{\mu'}{\mu}\right)^{n_1}\mathrm{Gr}^{n_2}\mathrm{Sc}^{n_3}\mathrm{K}_x^{n_4}\rho_{k0\infty}^{n_5}\mathrm{Re}_\mathrm{T}^{n_6}. \tag{20}$$

Equation (20) has a clear physical meaning. The various terms on the right side characterize the effect of individual factors on mass transfer in the gas phase: Gr — natural convection, Sc — the transport properties of the gas mixture, $\rho_{\mathrm{k}0\infty}$ — the concentration of uncondensed gas, $\mathrm{K_X}$ — the kinetics of the chemical reaction, $\mathrm{Re_T}$ — the removed heat flux, and μ'/μ — the friction on the phase interface. The Nusselt diffusion number is determined through the mass-transfer coefficient for the uncondensed component of the gas mixture. If we define

$$\beta_k = \frac{\rho D_k \nabla\rho_{k0}|_\mathrm{b}}{\rho_{k0\mathrm{b}}-\rho_{k0\infty}}, \tag{21}$$

then from the impermeability condition we obtain an equation for the mass flux in a form slightly different from the usual one:

$$j = \frac{\beta_k(\rho_{k0\mathrm{b}}-\rho_{k0\infty})}{\rho_{k0\mathrm{b}}}. \tag{22}$$

Although the direct proportionality between the quantities j and β_k is not so obvious in Eq. (22), a more thorough analysis shows that the mass-transfer coefficient defined in this way correctly reflects the quantitative aspect of mass transfer during condensation. For instance, an increase in the mass-transfer coefficient of the uncondensed component always leads to an increase in mass flux across the phase interface and vice versa. We should add that although β_k, by definition, characterizes only the diffusion transport of the uncondensed component, the quantity j in the equation is the total flux of uncondensed substance (diffusion and convective).

Such an approach greatly simplifies the analysis of mass transfer during condensation of multicomponent gas mixtures with chemical reactions. For instance, in the case of condensation of the system

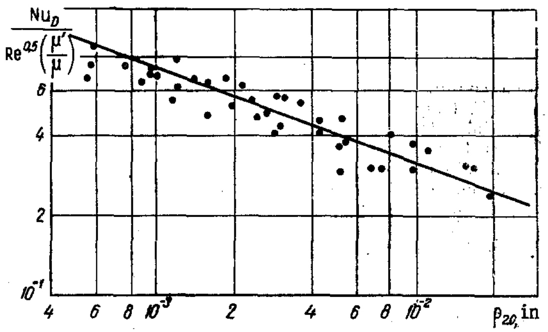

Fig. 2. Correlation of experimental data of [7] for condensation of steam–air mixture on a horizontal tube.

$N_2O_4 \rightleftharpoons 2NO_2 \rightleftharpoons 2NO + O_2$ difficulties are encountered in determination of the total diffusion flux of condensed N_2O_4 and NO_2 molecules on the phase interface. The relation between the concentration gradients of these components is very complex and the fluxes, expressed in the Fick form, are usually opposite in direction.

Using the dimensionless equation (20) we processed the experimental data of Kovalev et al. [2] for condensation of the heat-transfer medium N_2O_4 of nonequilibrium composition on a horizontal tube in the presence of the uncondensed, but mutually reacting, components NO and O_2 [2]. Treatment of these experimental data by the method proposed in the present work gave the following dimensionless equation, which correlates the experimental results with a root-mean-square error of 8%:

$$Nu_{D_x} = 0.88 \cdot 10^{-3} Ar^{0.25} Sc^{0.75} K_x^{0.2} Re_T^{0.5} \rho_{40_\infty}^{-0.41} \left(\frac{\mu'}{\mu}\right). \tag{20a}$$

Figure 1 shows the complex $A = Nu_{D_x}/Ar^{0.25}Sc^{0.75}K_x^{0.2} \times \rho_{40_\infty}^{-0.41}(\mu'/\mu)$ as a function of the "thermal" Reynolds number. As Fig. 1 reveals, the greatest values of Nu_{D_x} are obtained at high pressures, which is perfectly consistent with current ideas on the mechanism of the chemical reaction in an N_2O_4 system [6].

In Eq. (20a) the index of the power of ρ_{40_∞} not only takes into account the effect of the amount of uncondensed gases, but also, in implicit form, the order of the chemical recombination reaction. A more general expression for the case of a reaction of any order is obtained by introduction into the complex K_x of the intensity of a mass source (sink) of the k-th component,

$$K_x' = \frac{I_k d_0^2}{\rho D_k}. \tag{23}$$

In particular, for the considered system the index of the power of ρ_{40_∞} in the general equation will vary from −0.41 to −1.0, and the constant will be decreased by a factor of 1.32.

A paper dealing with the processing of experimental data for the condensation of N_2O_4 of nonequilibrium composition on a horizontal tube, obtained in the Institute of Nuclear Power, Academy of Sciences of the Belorussian SSR, has recently been published. The data were processed by using Eq. (20a), except that in calculation of the complex Re_T the superheating was taken into account, and the simplex μ'/μ was omitted in view of the insignificant change of this parameter. Inclusion of superheating led to a change in the index of the power of ρ_{40_∞}. Otherwise, the results of [8] agree satisfactorily with the data presented in Fig. 1.

Equation (20) can also be used to correlate the experimental data for the condensation of nonreacting vapor–gas mixtures. In this case K_x is omitted from the set of dimensionless numbers.

Isachenko and Bogorodskii investigated experimentally film and dropwise condensation of water vapor from a steam–air mixture on a horizontal tube [7]. The authors carried out two series of experiments. In the first series of experiments the authors failed to secure a symmetric feed of vapor–gas mixture, which prevented a consideration of natural convection in pure form. In fact, in processing the experimental results the authors found that the introduction of Gr (or the numerically equal Ar) into the dimensionless equation did not reduce but, on the contrary, increased the spread of the points relative to the correlating line.

Treatment of these results by using Eq. (20) also showed that introduction of Ar or Gr made the correlation less accurate, but the introduction of the simplex μ'/μ (Fig. 2) reduced the spread of the experimental points. The experimental data were correlated with a root-mean-square error of 5.3% by the equation

$$\mathrm{Nu}_D = 0{,}061\mathrm{Re}_{\mathrm{T}}^{0{,}5}\rho_{20,\,\mathrm{in}}^{-0{,}36}\left(\frac{\mu'}{\mu}\right). \tag{20b}$$

In conclusion, we express our thanks to B. S. Petukhov, S. A. Kovalev, A. S. Komendantov, V. P. Isachenko, and A. S. Bogorodskii for kindly providing the experimental data.

NOTATION

r, s, curvilinear coordinates; w_r, w_s, velocity components; g, gravitational acceleration; ν, kinematic viscosity; T, temperature; a, thermal diffusivity; ρ_{k0}, relative density of k-th component in mixture; I_k, sink (source) of mass of k-th component due to chemical reaction; T_0, tube wall temperature; T_s, temperature on vapor–liquid boundary; μ, dynamic viscosity; λ, thermal conductivity; h, enthalpy; δ_T, δ_d, δ_h, thicknesses of thermal, diffusion, and hydrodynamic boundary layers, respectively; $d_b = d_0 + 2\delta'$; d_0, outer diameter of tube; W_r, W_s, maximum values of velocity components; q, heat flux density; K_{rec}, recombination constant of reaction $O_2 + 2NO \rightleftharpoons 2NO_2$; D_k, diffusion coefficient of k-th component in mixture; M, molecular weight; β_k, mass-transfer coefficient. Dimensionless numbers:

$$\mathrm{Re}_{\mathrm{T}} = \frac{qd_0}{\Delta h\mu'}\,;\quad \mathrm{Nu}_D = \frac{\beta_k d_0}{D_k}\,;\quad \mathrm{Sc} = \frac{\nu}{D_k}\,;\quad \mathrm{Gr} = \frac{g\beta\Delta t d_0^3}{\nu^2}\,;$$

$$\mathrm{K}_x = \frac{\mathrm{K}_{\mathrm{rec}}\rho^2 d_0^2}{m_k^2 D_k}\,;\quad \mathrm{K}_x' = \frac{I_k d_0^2}{\rho D_k}\,.$$

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